2018-00652 - Post Doctoral Position about Verified Properties of Special Functions

Contract type : Public service fixed-term contract
Level of qualifications required : PhD or equivalent
Function : Post-Doctoral Research Visit

Context
This postdoc is partly funded by Atlantic 2020 (Région Pays de Loire). The aim of this project is to investigate and develop verified computer-aided proofs of properties of special functions, such as numerical values, symmetries, expansions, etc. Handbooks of special functions, like for instance the one edited by the NIST, contain well-known formulas as well as cutting edge results about the functions used in many application areas like physics, engineering but also combinatorics, number theory, etc. They are among the most cited references in the mathematical literature. The goal of this project is to start a small and formally verified handbook of special functions, using the Coq proof assistant. The building blocks of this work will be useful to other applications, like verified numerical approximation, verified quadrature, etc.

This project is related to the ANR Project Fastrelax and will take place in collaboration with members of the Fastrelax team. Travel expenses for conferences and collaborations are covered within the limits of the scale in force.

Assignment
The goal of this post-doc is to design formal libraries about so-called D-finite functions, i.e. solutions of linear differential equations with polynomial coefficients. This well-delimited class of functions includes a significant proportion of the entries of the aforementioned handbooks and has been thoroughly studied from an algorithmic point of view. This corpus of computer algebra algorithms provides systematic and computational ways to obtain properties of D-finite functions, and can even be used to generate dynamical dictionaries like the Dynamic Dictionary of Mathematical Functions.

These formal libraries will be written with and for the Coq proof assistant. Providing a formally verified version of this corpus requires significant extensions of the existing formal libraries about real and complex analysis. These extensions include the elementary theory of linear differential equations such as formal series, existence theorems and expansions at singularities. The other component of this work is the implementation of verified symbolic computations about D-finite functions. The Gfun Maple package provides a tour of the possible directions. A first benchmark will consist in connecting this work with existing libraries about numerical approximations via Taylor series and numerical integration, so as to extend the modularity and the capabilities of these verified and rigorous approximation tools. A longer term objective is to provide formally verified entries of generated dictionaries.

Main activities
- Study the computer algebra literature about D-finite functions
- Review the classical litterature in real and complex analysis, with a formalization perspective
- Develop verified implementations and formal proofs of these algorithms and results
- Evaluate the performance of these implementations
- Write reports and submit research articles

Skills
The topic of this project is at the intersection of computer algebra and formal proofs. A strong taste for these two fields is required, and in particular for the implementation of formal libraries. The candidate should hold a PhD in computer algebra or in logic.

Benefits package
- Subsidised catering service
- Partially-reimbursed public transport
- Social security
- Paid leave
- Flexible working hours
- Sports facilities