For the solution of large linear systems of the form $Ax = b$ with an agnostic lossy data compression approach, iterative solvers such as GMRES while preserving convergence. Empirically, the SZ can be used to checkpoint some strictly user set error controls. The team has shown that achieving very high compression ratios while respecting the extreme resilience team of the Mathematics and Computer Science division at Argonne National Lab is currently developing a comprehensive effort for lossy compression for scientific data in the context of the US Exascale Computing Project (ECP). In particular, the team has developed the SZ lossy compressor \([3, 6]\) that has been shown to enter an area is granted by the laboratoire. The joint project will study how lossy compression can be monitored by Krylov solvers to create jobs.

### Context

This position is open in the framework of the Joint Laboratory for Extreme Scale Computing (JLESC) within a collaboration between Inria and Argonne national laboratory. The joint project will study how lossy compression can be monitored by Krylov solvers to significantly reduce the memory footprint when solving very-large sparse linear systems. The resulting solvers will alleviate the I/O penalty paid when running large calculations using either check-point mechanisms to address resiliency or out-of-core techniques to solve huge problems.

### Assignment

For the solution of large linear systems of the form $Ax = b$ with an agnostic lossy data compression approach, iterative solvers such as GMRES while preserving convergence.
where $A \in \mathbb{R}^{n \times n}$, $x$ and $b \in \mathbb{R}^n$, Krylov subspace methods are among the most commonly used iterative solvers; they are further extended to cope with extreme scale computing as one can integrate features such as communication hidden in their variants referred to as pipelined Krylov solvers [2]. On the one hand, the Krylov subspace methods such as GMRES allow some inexactness when computing the orthonormal search basis; more precisely theoretical results [4, 5] show that the matrix-vector product involved in the construction of the new search directions can be more and more inexact when the convergence towards the solution takes place. An inexact scheme of that form writes into a generalized Arnoldi equality

$$[(A + E_1)v_1, \ldots, (A + E_k)v_k] = [v_1, \ldots, v_k, v_{k+1}] H_k.$$ (1)

where the theory gives a bound on $\|E_k\|$ that depends on the residual norm $\|b - Ax_k\|$ at step $k$, where $x_k$ is the $k$th iterate. Such a result has a major interest in applications where the matrix is not formed explicitly, e.g., in the fast multipole (FMM) or domain decomposition (DDM) methods context, where this allows one to drastically reduce the computational effort.

One the other hand, novel agnostic lossy data compression techniques are studied to reduce the I/O footprint of large applications that have to store snapshots of the calculation, for a posteriori analysis, because they implement out-of-core calculation or for checkpointing data for resilience. Those lossy compression techniques allow for precise control on the error introduced by the compressor to ensure that the stored data are still meaningful for the considered application. In the context of the Krylov method, the basis $V_{k+1} = [v_1, \ldots, v_k, v_{k+1}]$ represents the most demanding data in terms of memory footprint, so that, in a fault-tolerant or out-of-core context, storing it in a lossy form would allow for a tremendous saving.

The objective of this postdoc is to dynamically control the compression error of $V_{k+1}$ to comply with the inexact Krylov theory. The main difficulty is to translate the known theoretical inexactness on $E_k$ into a suited lossy compression mechanism for $v_k$ with loss $\|\delta v_k\|$.  

Main activities

The successful candidate will share her/his time between Inria Bordeaux and Argonne National Laboratory to work on the activities that will follow the tentative agenda given below:

- **M0-M2 at Inria**: theoretical analysis to translate the perturbation control from $\|E_k\|$ into a computable norm perturbation control on $\|\delta v_k\|$ (3 months).
- **M3-M6 at Argonne**: design/tune a lossy compression technique so that the loss will be below $\|\delta v_k\|$ (3 months).
- **M7-M9 at Inria**: implement/integrate the compression technique into a parallel out-of-core GMRES solver to evaluate the gain on large problems (4 months).
- **M10-M15 at Inria**: extend the methodology to block pipelined Krylov techniques [2] for the solution of linear systems with multiple right-hand sides [1] (6 months).

Benefits package
- Subsidised catering service
- Partially-reimbursed public transport

**Remuneration**

2653€ / month (before taxes)