Offer #2023-06934

Stage M2 - Internship / Bayesian Inverse Problems for Wave Propagation towards Exascale

Contract type: Internship agreement

Level of qualifications required: Master's or equivalent

Fonction: Internship Research

Context

In the framework of the ExaMA project within the NumPEx pro-gram (https://numpex.irisa.fr/), we propose a 5 months internship on the topic of Ensemble Methods for Bayesian Inverse Problems. The objective of NumPEx is to develop an exascale software stack, and the ExaMA project is dedicated to the mathematical algorithms to achieve this. The Inria MAKUTU project specializes in inverse problems for wave propagation problems and the theoretical and numerical methods needed for their solution.

Assignment

In an inverse problem, we seek the (unknown) model parameters from (known) measurements of the system’s solution. The direct system — in our case a partial differential equation with its boundary and initial conditions — can be considered as an operator from parameter space into data/observation space. The inverse problem, from data space into parameter space, is ill-posed [1, 2]. The deterministic approach for solving the inverse problem consists of minimizing a cost function that expresses the error, or mismatch, between the model predictions and the measured observations, or data. We usually add a regularization term (Tikhonov approach) to penalize solutions that are too oscillatory. This nonlinear minimization is usually performed by a method from the quasi-Newton family [3]. Bayes’ Theorem provides a framework for the solution of a more general inverse problem, where we seek the posterior probability distribution of a function, given prior knowledge of its distribution and measurements of an observed quantity that depends on the unknown, or hidden function. Mathematically, the theorem quantifies a posterior probability distribution (ppd) as a function of an a priori distribution that captures any previous knowledge about the parameters, and a likelihood function that is obtained by solving the direct model for given parameter values. The resulting a posteriori law is both the solution of the inverse problem and provides a complete quantification of the uncertainty in this solution. The numerator, being the product of two positive terms, less than one, implies that the result of Bayesian inversion produces a reduction of the uncertainty that gives probability distributions with smaller variance. The posterior distribution is usually simulated using Markov Chain Monte Carlo (MCMC) methods [2, 4].

Main activities

In this internship, we want to study, both theoretically and numerically, the solution of an inverse problem for wave propagation. A simple case is described by the acoustic wave equation, in a one-dimensional domain, with varying sound speed function over the domain. This case can be easily generalized to more realistic wave propagation problems.

1. Based on [5], formulate the Bayesian inverse problem (BIP) for the wave equation.
2. Give all details of the existence, and convergence properites of the BIP solution.
3. Propose a MCMC method [2, 4] for solving the BIP.

An alternative approach for solving the BIP, is to use an ensemble Kalman filter (EnKF) [1, 2]. This method is known as ensemble Kalman inversion, and was first formulated in [6].

4. Based on [6], formulate the EKI approach for the wave equation.
5. Give all details of the existence, and convergence properites of the EKI solution.
6. Propose an EnKF method [2] for solving the EKI problem. Both MCMC and EKI are methods that can be readily parallelized. This is the ultimate goal of the ExaMA project: to develop a highly parallelized library for the solution of inverse problems in wave propagation.

References

Skills
The candidate should have strong skills in applied mathematics for partial differential equations as well as some knowledge of probability and stochastic differential equations. The internship also requires numerical skills and some coding experience to solve the underlying partial differential equations and to implement the stochastic simulation methods. The candidate should be at ease in English, including reading, understanding and communication. The internship will be co-supervised by Prof. Mark Asch, who has a long experience in data assimilation and inverse problems.

Benefits package
- Subsidized meals
- Partial reimbursement of public transport costs
- Professional equipment available (videoconferencing, loan of computer equipment, etc.)

General Information
- **Theme/Domain**: Earth, Environmental and Energy Sciences
  Scientific computing (BAP E)
- **Town/city**: Pau
- **Inria Center**: Centre Inria de l’université de Bordeaux
- **Starting date**: 2024-03-01
- **Duration of contract**: 6 months
- **Deadline to apply**: 2024-01-31

Contacts
- **Inria Team**: MAKUTU
- **Recruiter**: Barucq Helene / Helene.Barucq@inria.fr

About Inria
Inria is the French national research institute dedicated to digital science and technology. It employs 2,600 people. Its 200 agile project teams, generally run jointly with academic partners, include more than 3,500 scientists and engineers working to meet the challenges of digital technology, often at the interface with other disciplines. The Institute also employs numerous talents in over forty different professions. 900 research support staff contribute to the preparation and development of scientific and entrepreneurial projects that have a worldwide impact.

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Instruction to apply

Please send your CV and cover letter to.

Defence Security:
This position is likely to be situated in a restricted area (ZRR), as defined in Decree No. 2011-1425 relating to the protection of national scientific and technical potential (PPST). Authorisation to enter an area is granted by the director of the unit, following a favourable Ministerial decision, as defined in the decree of 3 July 2012 relating to the PPST. An unfavourable Ministerial decision in respect of a position situated in a ZRR would result in the cancellation of the appointment.

Recruitment Policy:
As part of its diversity policy, all Inria positions are accessible to people with disabilities.