Offer #2025-09207

PhD Position F/M Filippov Solutions for Discontinuous Differential-Algebraic Equations (DAEs): Control and Simulation

Contract type : Fixed-term contract

Level of qualifications required : Graduate degree or equivalent

Fonction : PhD Position

About the research centre or Inria department

The Inria Rennes - Bretagne Atlantique Centre is one of Inria's eight centres and has more than thirty research teams. The Inria Center is a major and recognized player in the field of digital sciences. It is at the heart of a rich R&D and innovation ecosystem: highly innovative PMEs, large industrial groups, competitiveness clusters, research and higher education players, laboratories of excellence, technological research institute, etc.

Context

Collaboration & Contacts

- This PhD is conducted in collaboration with Prof. Stephan Trenn at the University of Groningen (The Netherlands). The selected candidate is expected to visit Groningen regularly.
- There is also the possibility of pursuing a double doctoral degree, which would include a 1-year extension to meet Dutch PhD requirements.
- This position will be funded by ANR JCJC Project: **GFdDAE** (ANR-25-CE48-4916)
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Context & Background

Differential-algebraic equations (DAEs) arise naturally when modeling dynamical systems from first principles. In many cases, physical laws are expressed as combinations of differential and algebraic equations. This modeling approach is common in constrained mechanics, chemical and biological processes, power systems, and especially analog circuit design—where idealized components (e.g., resistors, capacitors, inductors) and Kirchhoff’s laws define the system dynamics.

When these systems experience abrupt changes—such as switching in electric circuits, mechanical contacts, or discontinuous control inputs—**discontinuous DAEs** emerge. However, there is currently no comprehensive theoretical foundation for studying such systems. Challenges include:

- Their **hybrid behaviors**, which differ significantly from ODE counterparts,
- The **inconsistent initialization problem** caused by switching and algebraic constraints,
- The occurrence of **Dirac impulses** due to state jumps.

Without a rigorous solution concept, tasks such as simulation, stability analysis, and control design lack solid justification.

Discontinuous DAEs are relevant across many research areas, including systems and control, hybrid systems, and computer-aided simulation. A notable example is **switched DAEs**. While time-dependent switching has been extensively studied [2-5], progress on **state-dependent switching**, a subclass of discontinuous DAEs, remains limited.

The **Hycomes** team at Inria Rennes has contributed to related research through the concept of **multi-mode DAEs,** in the context of the **Modelica** language [6-7]. Despite these advancements, challenges persist, including:

- Computing consistent initial values and jumps,
- Managing sliding and chattering behaviors,
- Addressing scalability for large-scale, high-dimensional systems.

These issues emphasize the need for refined mathematical foundations and advanced control methods compatible with Modelica-based simulation platforms.

[1] D. Liberzon. Switching in Systems and Control. Systems and Control: Found. and Appl. Boston: Birkhäuser, 2003.

[2] D. Liberzon and S. Trenn. “On stability of linear switched differential algebraic equations”. In: Proc. IEEE CDC 2009, pp. 2156–2161.

[3] D. Liberzon and S. Trenn. “Switched nonlinear differential algebraic equations: Solution theory, Lyapunov functions, and stability”. In: Automatica 48.5, pp. 954–963.

[4] Y. Chen and S. Trenn. “Impulse-free jump solution of nonlinear differential algebraic equation”. In: Nonlinear Analysis: Hybrid Systems 46 (2022), p. 101238.

[5] Y. Chen and S. Trenn. “On impulse-free solutions and stability of switched nonlinear differential–algebraic equations”. In: Automatica 156 (2023), p. 111208

[6] A. Benveniste, B. Caillaud, and M. Malandain. “The mathematical foundations of physical systems modeling languages”. In: Ann. Rev. in Control 50 (2020), pp. 72–118.

[7] A. Benveniste, B. Caillaud, M. Malandain, and J. Thibault. “Algorithms for the structural analysis of multimode modelica models”. In: Electronics 11.17 (2022),

p. 2755.

[8] A.F. Filippov. Differential Equations with Discontinuous Right-hand Sides. English (Transl. from the Russian). Mathematics and Its Applications: Soviet Series, 18. Dordrecht etc.: Kluwer Academic Publishers, 1988.

[9] Y. Shtessel, C. Edwards, L. Fridman, A. Levant, et al. Sliding Mode Control and Observation. Vol. 10. Springer, 2014

Assignment

For discontinuous ODEs, the **Filippov solution framework** [8] plays important roles both theoretically (e.g., in switching ODE systems [1] and sliding mode control [9]) and practically (e.g., via Filippov-type solvers in MATLAB). This PhD project aims to:

1. Extend the **Filippov solution concept** to discontinuous DAEs,
2. Integrate the proposed framework into **simulation tools**, particularly **Modelica**.

Main activities

The PhD student will focus on the following tasks:

1. Conduct a thorough literature review of discontinues DAEs and related systems (e.g., complementarity systems, switching DAEs, hybrid systems).
2. Starting with **discontinuous linear DAEs**, propose a solution concept and prove **well-posedness**.
3. Extend the theory to **nonlinear systems** and compare it with other existing frameworks.
4. Perform **stability analysis** and develop **stabilization or control strategies** for discontinuous DAEs.
5. Implement simulation methods in **Modelica tools**, and test them on benchmark examples.

Deliverables include scientific reports, papers submitted to international conferences and journals, and prototype simulation code.

Skills

Technical Skills

- Solid understanding of **system modeling**, **control theory**, and **differential equations**
- Familiarity with any of the following topics would be appreciated:
  - **DAEs**, **switched systems**, **complementarity systems**, **sliding mode control**;
  - **hybrid systems simulation, Modelica**;
  - power electronics, **contact mechanics, multiphysics modeling**.

Benefits package

- Subsidized meals
- Partial reimbursement of public transport costs
- Possibility of teleworking (90 days per year) and flexible organization of working hours
- Partial payment of insurance costs

Remuneration

monthly gross salary 2200 euros

General Information

- **Theme/Domain :** Optimization and control of dynamic systems
  Information system (BAP E)
- **Town/city :** Rennes
- **Inria Center :** Centre Inria de l'Université de Rennes
- **Starting date :** 2026-01-02
- **Duration of contract :** 3 years
- **Deadline to apply :** 2025-09-22

Contacts

- **Inria Team :** HYCOMES
- **PhD Supervisor :**
  Chen Yahao / yahao.chen@inria.fr

About Inria

Inria is the French national research institute dedicated to digital science and technology. It employs 2,600 people. Its 200 agile project teams, generally run jointly with academic partners, include more than 3,500 scientists and engineers working to meet the challenges of digital technology, often at the interface with other disciplines. The Institute also employs numerous talents in over forty different professions. 900 research support staff contribute to the preparation and development of scientific and entrepreneurial projects that have a worldwide impact.

The keys to success

Requirements

- M.Sc. degree in **systems and control**, **applied mathematics**, **computer science**, or related fields
- Proficiency in **academic English writing** and fluency in **spoken English**
- Strong **mathematical thinking** and problem-solving skills
- Scientific curiosity, **autonomy**, and the ability to work independently

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Instruction to apply

Please submit online : your resume, cover letter and letters of recommendation eventually

Defence Security :
This position is likely to be situated in a restricted area (ZRR), as defined in Decree No. 2011-1425 relating to the protection of national scientific and technical potential (PPST).Authorisation to enter an area is granted by the director of the unit, following a favourable Ministerial decision, as defined in the decree of 3 July 2012 relating to the PPST. An unfavourable Ministerial decision in respect of a position situated in a ZRR would result in the cancellation of the appointment.

Recruitment Policy :
As part of its diversity policy, all Inria positions are accessible to people with disabilities.