A propos du centre ou de la direction fonctionnelle

The scientific corpus shared by the members of the team Factas involves functional and harmonic analysis, along with related fields like approximation theory, potential theory, Schur analysis or system and circuit theory. Our approach is to couple theoretical tools from these fields with constructive optimisation techniques, and to demonstrate efficiency of such techniques in selected application areas. The latter mainly comprises microwave electronics, notably the synthesis and tuning of circuitual devices for communication, as well as inverse problems in quasi-static electromagnetism, with applications to paleomagnetism in planetary sciences and to imaging issues in electro- and magnetoencephalography (EEG and MEG for short) for functional and medical neurosciences.

We try to balance theoretical and applied work without sacrificing any of them, a transverse positioning made possible by the complementarity of skills within the team, ranging from mathematical analysis to algorithm design and numerical optimisation, software design as well as some knowledge in microwave electronics, brain imaging and magnetometry. In all cases, the overall objective is the production of prototypical software tools for the user, dedicated to the application at hand.

Contexte et atouts du poste

The Inria Sophia Antipolis – Méditerranée center counts 37 research teams and 9 support departments. The center's staff (about 600 people including 400 Inria employees) is composed of scientists of different nationalities (250 foreigners of 50 nationalities), engineers, technicians and administrative staff. The majority of the research teams at the center are located in Sophia Antipolis and Nice in the Alps-Maritimes. Six teams are based in Montpellier and a team is hosted by the computer science department of the University of Bologna in Italy. The Center is a member of the University and Institution Community (ComUE) “Université Côte d’Azur (UCA)”.

Mission confiée

Context

ElectroEncephaloGraphy (EEG) and MagnetoEncephaloGraphy (MEG) are the only two modalities that allow to register the electrical activity of the human brain with a temporal resolution compatible with its working speed. EEG and MEG are used for the diagnosis of pathologies such as epilepsy, and are distinguished techniques in cognitive neurosciences in order to examine and to understand brain functions. For a long time, EEG analysis consisted in the study of rythm and in coarse localization of events taking place next to the sensors, hence at the surface of the scalp. The recent introduction of MEG, together with the advent of new magnetic recording techniques (increased number of sensors for EEG and new features thereof for MEG), and the exploitation of biophysical techniques, allow one to consider the localization of sources of activity and to foresee connectivity studies between the various identified regions, in order to better understand the functioning or dys-functioning of the brain. These localization techniques amount to solving an inverse problem which has been the topic of many developments, although they remain far from completion and still suffer lack of validation.

The inverse problem involves a temporal dimension and a geometrical dimension. This subject is mostly dedicated to the latter, which makes for an inverse harmonic potential problem whose source term in divergence form, and the PhD thesis aims at developing new regularization methods, suitable for the study of foci of the brain electrical activity, that the team Factas at INRIA will consider jointly with the team Dynamap at INS.

From the mathematical point of view, we face an inverse problem for the Poisson-Laplace equation: 
\[ \Delta u = \text{div} \, \mu \] , where the source term \( \mu \) is to be determined as a \( \mathbb{R}^3 \)-valued measure (or generalized function), from pointwise measurements of the potential U (EEG) or components of the magnetic field B (MEG), taken in a region or a surface located at some distance from the support of \( \mu \), see [5,7].

This type of inverse problems appears in diverse contexts with various geometries, typically for models governed by Maxwell's equations in quasi-static regime. As regards neurosciences, in EEG or MEG, the source distribution \( \mu \) represents the primary cerebral current, often modelled by a finite linear combination, with coefficients in \( \mathbb{R}^3 \), of pointwise Dirac masses located within the brain or distributed on the cortex. This justifies the choice of the unknown \( \mu \) as a measure.

In EEG, measurements of the electrical potential U at electrodes laid on a subregion of the scalp are available. In MEG, measurements of the normal component of the magnetic field are taken by ultra sensitive magnetometers and gradiometers that use superconducting coils displayed on a helmet at some (small) distance from the scalp.

In the case of EEG, the medium conductivity actually acts multiplicatively on the equation, and because scalp, skull, and brain possess different conductivity values, one often applies a preliminary...
data transmission step, so-called cortical mapping, in order to compute the potential at the surface of the cortex, which is homogeneous, and to end up with a Poisson equation. In the case of MEG with spherical layered head models, the magnetic propagation of the radial component of $B$ is in principle insensitive to the conductivity, and its curl is proportional to $\mu$, as for an homogeneous medium. Note that, outside the head, $B$ is the gradient of some harmonic scalar potential.

Initial measurements are incomplete, since they are pointwise and limited in number, moreover they concentrate on a strict subset of the scalp (EEG) or of a virtual surface encompassing the head and containing the helmet (MEG). The smoothness of this surface and of the surface of the cortex of course plays a role in algorithms dedicated to these problems. Identifiability issues, in particular, are different depending on the measurements, and sources may or may not be located in geometrically separated sets (in the present situation, the non-separated case occurs if, after data transmission, we look for a distribution supported on the surface of the cortex); such issues also depend on the smoothness of the set in which the support of $\mu$ is assumed to lie. In particular, the numerical modelling of the surface of the cortex and its approximation by a sphere o an ellipsoid, or yet another analytic surface, induce different algorithmic choices.

If measured or estimated values of the potential or the field are known on a surface, we typically want to approximate them in quadratic norm by a term of type $\mu_A$, where $A$ is the forward operator mapping $\mu$ to the available measurements of $U$ or $B$ (which is linked to the leadfield matrix if $\mu$ consists of pointwise masses and if the measurements are taken at electrodes for EEG or at sensors for MEG). However, its ill-posedness makes it necessary to regularize this optimization problem by adding a penalization, weighing on some features of $\mu$, see for instance [1].

A discretization is frequently applied before any optimization step is carried out, and usually only discrete models are considered, which are fully described by the leadfield matrix. One drawback of this approach lies with the resulting mix-up between the regularizing effect of the discretization (uncontrolled, in general), and the one provided by the constraint in the optimization process. In particular, alterations induced on the kernel of $A$ by the discretization impinge on the behaviour of solutions, their stability and sparsity properties. In recent years, for the discrete case, $l^1$ minimization methods with $l^1$ penalization of the variable became very popular to solve inverse problems, and came to be known as compressed sensing, see [6]. The gist of these methods is to favor the determination of sparse solutions, when they exist. We aim at developing a similar approach in the present setting that accounts for the infinite dimensional character of the underlying model.

**Topic of the thesis**

The goal of the thesis is to develop a continuous version of compressed sensing, appropriate for the present framework, where sparsity could be defined through the dimension of the support of the (vector valued) measure $\mu$, which represents the primary cerebral current for EEG and MEG. Here, the decomposition in elementary solenoids of divergence-free measures plays a key role in the structure of the kernel of the forward operator $A$. The analog of a $l^1$ constraint on the measure $\mu$ is the so-called total variation of $\mu$, see [2].

The thesis will consider solving, in the continuous setting, $l^1$ optimization problems on $\mu$ under constraint on its total variation, together with algorithms for the approximation of solutions suitable in the context, inspired for example from [4]. Convergence and consistency issues will be examined, especially in the case of sparse sources and noisy data. Note that we discretize here a posteriori, and not a priori. Such algorithms could, in some cases, be initialized from results obtained with the software FindSources3D [FS3D], http://www-sop.inria.fr/kips/FindSources3D/.

Observe that there are other applications to non-destructive control of inverse source problems to be studied in the thesis, among which geomagnetism [3] and paleomagnetism, which also constitute research topics of the team Factas. Thus, we expect some synergy to take place between techniques from various domains.

On the experimental side, our medical partners at INS enjoy highly performing measurement devices, which brings an important applied side to the subject, since real data are available. Indeed, the team Dynamap of INS manages a MEG platform. It is one of the few teams worldwide able to perform simultaneous recordings in MEG and in stereo EEG (SEEG), with deep electrodes. This last technique consists in recording structures within the brain of epileptic patients in order to identify regions responsible for the seizures and to suggest curative surgeries.

Such datasets are the only ones that allow us to validate localizations or treatments performed simultaneously by non invasive EEG and MEG recordings. Data and results exchange protocols are being set between the teams Dynamap and Factas, especially using the softwares Anywave (http://meg.univ-amu.fr/wiki/AnyWave) and FS3D, in order to test the methods developed in the course of the PhD work on actual data, and to draw inspiration from experiments to enhance the proposed techniques.

Those will constitute the algorithmic and computational aspects of the PhD. In particular, comparisons of the results will be run, in terms of source localization depending on the source term properties (pointwise dipolar or distributed surface), on the data (synthetic or experimental, EEG or MEG or EEG+MEG or SEEG+MEG, where SEEG requires specific technique), and on the methods (used in distributed softwares like MNE, \(l^1\), or in FS3D / Anywave and other methods still to be developed).

More prescriptively, we also plan to take advantage of the next generation of magnetic sensors (OPM) that should make the 3 components of B available, for MEG studies.

The expertise of the two teams in these domains and the unique character of the available data provide a rare opportunity for the development and the validation of efficient and robust mathematical and algorithmical methods, important both from the clinical and on the fundamental neurosciences viewpoints.

Principales activités
Research.

Compétences
Good knowledge in mathematical analysis and acquaintance with Matlab. Interest for applications in physics and medical imaging.

Avantages
- Subsidized meals
- Partial reimbursement of public transport costs
- Leave: 7 weeks of annual leave + 10 extra days off due to RTT (statutory reduction in working hours) + possibility of exceptional leave (sick children, moving home, etc.)
- Possibility of teleworking (after 6 months of employment) and flexible organization of working hours
- Professional equipment available (videoconferencing, loan of computer equipment, etc.)
- Social, cultural and sports events and activities
- Access to vocational training
- Social security coverage

Rémunération
Duration: 36 months
Location: Sophia Antipolis, France
Gross Salary per month: 1982€ brut per month (year 1 & 2) and 2085€ brut/month (year 3)