In 2019, we developed general probabilistic criteria for exponential convergence of conditional distributions of absorbed processes to quasi-stationary distributions (QSDs), which extends the notion of stationary distributions in cases where stationary distributions may not exist. Most of the time, these models are obtained as scaling limits of discrete/microscopic models.

In many macroscopic deterministic or stochastic models, extinction or absorption may only arise in the limit of infinite time in which case stationary distributions may not exist. Most of the time, these models are obtained as scaling limits of discrete/microscopic models.

Recently, we developed general probabilistic criteria for exponential convergence of conditional distributions of absorbed processes to QSDs based on coupling techniques [1,3]. However, several methodological approaches developed in the classical context of stationary (non-absorbed) processes remain poorly understood in the absorbed case. This is typically the case of questions of convergence and approximation of QSD when the underlying process converges to some limit process, which may be deterministic or stochastic, and absorbed or non-absorbed.

The post-doctoral project focuses on several open questions in this context.

Bibliography


Contexte et atouts du poste

Team

TOSCA, INRIA Nancy Grand-Est
https://team.inria.fr/tosca/

Contacts

Nicolas Champagnat (Nicolas.Champagnat@inria.fr) and Denis Villemonais (denis.villemonais@univ-lorraine.fr).

This post-doctoral project is proposed for Inria Subvention funding (https://www.inria.fr/centre/nancy/presentation/offres/post-doctorat/sejours-post-doctoraux).

Mission confiée

Context

The post-doc position will take place in the TOSCA team of Inria Nancy -- Grand Est and the laboratory of Mathematics of Université de Lorraine, Institut Elie Cartan de Lorraine (IECL).

Absorbed Markov processes are ubiquitous in a wide range of application domains, including populations dynamics (ecology, evolution, population genetics) where absorption corresponds to extinction of a (sub)population, distributed algorithms, where absorption corresponds to deadlocks, metastable dynamics, where one is interested on the exit time/position of an attracting subdomains (e.g. in molecular dynamics), nuclear physics (e.g. to quantify the amount of radioactive particles that exit from a nuclear reactor) and more generally in the study of transitory behaviours of stochastic systems (e.g. for problems of reliability, survival...).

The state of the system before absorption may be characterized by a so-called quasi-stationary distribution (QSD), which extends the notion of stationary distribution to absorbed processes. Up to now, most of the studies on QSD focus on their existence and uniqueness, and on large time convergence of conditional distributions, mostly based on spectral methods [3].

In many macroscopic deterministic or stochastic models, extinction or absorption may only arise in the limit of infinite time. Such models may either exhibit a stationary behaviour or an absorption in infinite time, in which case stationary distributions may not exist. Most of the time, these models are obtained as scaling limits of discrete/microscopic models.

Recently, we developed general probabilistic criteria for exponential convergence of conditional distributions of absorbed processes to QSDs based on coupling techniques [1,3]. However, several methodological approaches developed in the classical context of stationary (non-absorbed) processes remain poorly understood in the absorbed case. This is typically the case of questions of convergence and approximation of QSD when the underlying process converges to some limit process, which may be deterministic or stochastic, and absorbed or non-absorbed.

The post-doctoral project focuses on several open questions in this context.

Bibliography


**Principales activités**

**Project description**

The questions of convergence and approximation of QSD when the underlying process converges to a deterministic system converging to attracting point has already been studied in [4,5,6].

However, this question arise naturally in many other contexts where the limit process is stochastic: in population genetics, where a discrete population dynamics converges to a diffusion process with absorption; in molecular simulation using numerical approximation of diffusions (see [7] for an example where absorbed processes are of particular interest); in ecology where the influence of varying environmental conditions is often modeled as piecewise deterministic Markov processes (PDMP), which arise naturally as scaling limits of discrete processes. In all these applications, the stationary behaviour of the converging or limit processes is of particular interest.

The post-doctoral student will study the problem of convergence and approximation of a QSD by the QSDs of a converging sequence of process, considering various situations where the limiting process can be absorbed or non-absorbed. Typical problems will include diffusion approximations of birth-death processes, Euler scheme approximations of diffusions absorbed at the boundary of a domain, PDMP approximation and stochastic processes with adaptive cutoff on the absorption rate, with applications to non-classical Markov Chain Monte Carlo methods. The selected candidate will also study the implications of this convergence results on the quantification of convergence to equilibrium for meta-stable stochastic processes.

These questions will be addressed using the recent criteria to quantify the convergence to QSD developed in [1,2]. These questions are also related to problems of continuity of QSD when the underlying process depends on a parameter. The supervisors are currently developing tools for this topic, which will certainly be useful in this post-doctoral project. Another original question is related to this project: how to define properly a notion of quasi-stationary distribution for processes which go extinct in infinite time, i.e. how to condition a non-absorbed stochastic process to remain away from extinction for an infinite time? The postdoctoral fellow will also study the numerical validation and implications of these results, using particle approximation schemes [3].

The postdoctoral results will be the object of publications in peer-reviewed international journals and of communications in international conferences of the field. This work will pave the way to future collaborations with specialists both of numerical methods and of convergence to equilibrium of Markov processes.

**Compétences**

**Required qualifications**

PhD thesis in probability theory / stochastic processes.

Specific expertise on convergence of Markov processes and ergodicity criteria are welcome.

**Language**

French or English.

**Avantages**

- Subsidized meals
- Partial reimbursement of public transport costs
- Leave: 7 weeks of annual leave + 10 extra days off due to RTT (statutory reduction in working hours) + possibility of exceptional leave (sick children, moving home, etc.)
- Possibility of teleworking (after 6 months of employment) and flexible organization of working hours
- Professional equipment available (videoconferencing, loan of computer equipment, etc.)
- Social, cultural and sports events and activities
- Access to vocational training
- Social security coverage

**Rémunération**

Salary: 2653€ gross/month