In the field of automatic program optimization, iterative structures, or loops, are important targets, since they represent a significant part of the total execution time. This is the reason why many loop optimizing transformations have been proposed to make them faster. Such transformations are mostly addressing loop parallelization and the improvement of temporal and spatial data localities. They usually apply to a set of nested loops, called a loop nest. Examples of transformations are loop interchange, loop fusion, or loop tiling. The polyhedral model is a mathematical generalization of such loop transformations. It applies to loop nests where each loop has one unique loop index whose lower and upper bounds are either constant values, or affine functions of invariant parameters, or affine functions of surrounding loop indices and invariant parameters. Indices must vary following a constant step at each iteration. Moreover, memory references of instructions inside the loops can only be related to scalar variables or to static array elements referenced by affine functions of the surrounding loop indices and invariant parameters. However, the scope of the polyhedral model has been recently extended to the generation of conditionally optimized code and to runtime speculative parallelization.

In the polyhedral model, iterations of a loop nest are represented geometrically as points whose integer coordinates are the loop indices. Their convex hull is a convex polyhedron called the iteration domain. Its definition may depend on parameters, as it is for the loop bounds. Thus, a loop transformation is actually a geometrical transformation of its iteration domain. Every usual loop transformation can be expressed and applied using the polyhedral model.

However, such transformations are all linear, or quasi-linear. They may all be defined using linear transformation matrices of iteration domains, or using quasi-linear transformation functions that include integer divisions and modulos.

Although it has been showed in many works that (quasi-)linear transformations may be very efficient, they are still limited by their linear aspect. Indeed, we have shown that (1) the collapsing of non-rectangular loops and (2) loop tiling and parallelization with load balancing are very efficient optimizing non-linear transformations based on symbolic roots of multi-variate polynomials.

## Project description

The addressed algebraic optimizing transformations are based on the counting of integer points inside parametric polyhedra. The main underlying mathematical theory is related to Ehrhart polynomials.

Ehrhart polynomials are integer-valued polynomials that express the exact number of integer points contained by a finite polyhedron, i.e., a polytope, which depends linearly on integer parameters. If we consider a d-dimensional polytope depending linearly on parameters p₁, p₂, …, pₘ, then its Ehrhart polynomial is a polynomial of degree d whose variables are p₁, p₂, …, pₘ, and whose coefficients are periodic numbers. However, every coefficient is constant whether every vertex of the polytope has integer coordinates. Ehrhart polynomials can be computed automatically by using software implementations as the one in library barvinok.

Among interesting properties of loop nests, the order, or rank, of an iteration is given by an Ehrhart polynomial: given a tuple of possible values of the loop indices, the rank of the associated iteration is the number of iterations that are executed before

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**Application deadline**

April 30th, 2020 (Midnight Paris time)

**How to apply**

Upload your file on jobs.inria.fr in a single pdf or zip file, and send it as well by email to philippe.clauss@inria.fr.

Your file should contain the following documents:

- Your CV.
- A cover/motivation letter describing your interest in this topic.
- A short (max one page) description of your Master thesis (or equivalent) or of the work in progress if not yet completed.
- Your degree certificates and transcripts.
This number is the number of tuples in the iteration domain which are lexicographically less than the addressed tuple. Thus, a ranking Ehrhart polynomial associates to each tuple characterizing an iteration, a unique integer value between 1, which is the rank of the very first iteration, and the total iteration count of the loop nest. Conversely, one unique tuple of the iteration domain is associated to an integer value in the interval. Thus, a ranking polynomial defines a bijection, and can then be inverted.

We have developed a technique to invert such ranking polynomials that has been implemented in a dedicated software. The mathematical expressions that resulting are called Trahrhe expressions. They are algebraic expressions of complex numbers, including radicals, and from which are extracted their real and integer parts.

The first goals of the PhD work are to consolidate the existing developments:
- Improving the efficiency of non-rectangular loop collapsing and algebraic tiling;
- Improving and extending the software implementation for computing and applying Trahrhe expressions.

Then, the work should be oriented toward theoretical and practical extensions:
- Extending algebraic tiling to stencil computations requiring loop skewing;
- Developing new algebraic optimizations (e.g. algebraic scheduling);
- Generalizing algebraic transformations;
- Implementing an automatic chain of algebraic optimizations, for example as passes of the compiler Clang/LLVM.

Compétences

Required qualifications

MSc in computer science, good knowledge in compiler optimizations and parallel programming, good mathematical skills.

Language

Fluent English.

Avantages

- Subsidized meals
- Partial reimbursement of public transport costs
- Leave: 7 weeks of annual leave + 10 extra days off due to RTT (statutory reduction in working hours) + possibility of exceptional leave (sick children, moving home, etc.)
- Possibility of teleworking (after 6 months of employment) and flexible organization of working hours
- Professional equipment available (videoconferencing, loan of computer equipment, etc.)
- Social, cultural and sports events and activities
- Access to vocational training
- Social security coverage

Rémunération


Monthly salary after taxes: around 1596,05€ for 1st and 2nd year. 1678,99€ for 3rd year.

In addition, one recommendation letter from the person who supervises(d) your Master thesis (or research project or internship) should be sent directly by his/her author to philippe.clauss@inria.fr.

Applications are to be sent as soon as possible.

Consignes pour postuler

Sécurité défense:

Ce poste est susceptible d’être affecté dans une zone à régime restrictif (ZRR), telle que définie dans le décret n°2011-1425 relatif à la protection du potentiel scientifique et technique de la nation (PPST). L’autorisation d’accès à une zone est délivrée par le chef d’établissement, après avis ministériel favorable, tel que défini dans l’arrêté du 03 juillet 2012, relatif à la PPST. Un avis ministériel défavorable pour un poste affecté dans une ZRR aurait pour conséquence l’annulation du recrutement.

Politique de recrutement:

Dans le cadre de sa politique diversité, tous les postes Inria sont accessibles aux personnes en situation de handicap.

Attention: Les candidatures doivent être déposées en ligne sur le site Inria. Le traitement des candidatures adressées par d’autres canaux n’est pas garanti.